Numerical Study of Fluid Flow in Sucker Rod Pump Using Finite Element Method

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Abstract
In order to know something about the flow of the fluid in sucker rod pump and obtain guidance for deciding the pump’s size and structure design, the fluid flow was calculated by using the $\varepsilon-K$ two-equation model. Based on the finite method, we analyzed several situations in a pumping period.

Introduction
It is generally understood that different kinds of fluid flow are often dealt with, such as liquid moving to and fro in the pump, vertical or declining motion of single-phase and multiphase fluid in the oil pipe, and so on, when the sucker rod pumping system is working. Due to the complexity of pump’s inner structure and the reciprocation of liquid movement in the sucker rod pump, the problem of analyzing the fluid flow in a pump is very complex and not yet entirely understood. Recently, because the study of reciprocation of liquid movement in the sucker rod pump is infrequent and liquid movement law is not yet entirely understood, numerical analysis has been applied to the study of fluid flow in pump by the method of finite element analysis with the help of ANSYS.

In order to analyze the fluid flow in a sucker rod pump, several pump valve and piston locations in an oil pump circulation were selected to set up the computational models to analyze. In this paper, the movement of crude oil that lies in the valve channels and orbicular channels of the pump is analyzed carefully. As a consequence, the liquid movement law can be obtained in an oil pump circulation according to the previous several special states.

Due to the complexity of pump’s inner structure and tortuous liquid path, a turbulent model has been applied to the computer model in the paper. In fact, liquid space is three-dimensional and the valves of the pump are changeable when oil pump is working, as a result, the fluid state in the pump is not steady. But in this paper, considering the limit of memory requirements in computer and the actual computational capability of computer, a few assumptions are made on the basis of practical working conditions and the two-dimensional steady fluid flow method is applied.

Single-phase fluid flow theory
Most pipe fluid flow in oil engineering includes the calculation and analysis of liquid pressure along the fluid line. The variation of the liquid pressure can be figured out according to the conservation of energy between two points in the range of pipe. The equation of motion is

$$\frac{dp}{dl} = -\frac{g}{g_c} \rho \sin \beta + \frac{\rho v^2}{g_c} \frac{dv}{dl} - (\frac{dp}{dl})_f$$  (1)

in which \( p \), \( \rho \), \( \beta \), \( g_e \), \( v \), and \( l \) are liquid pressure, liquid density, declining pipe angle, conversion coefficient, liquid velocity and pipe length, respectively. For \( g_e \), the magnitude is 32.2.

The previous equation shows that pressure gradient often includes three parts: immobile pressure gradient, acceleration gradient, and frictional gradient. The Darcy-Weibath equation is often applied

\[
\frac{dp}{dl} = f \frac{\rho v^2}{2g_e d}
\]

in which \( f \) and \( d \) are friction coefficient and pipe diameter.

In this paper, the liquid is assumed to be incompressible, that is, liquid velocity along a given constant section is equivalent and potential energy is also equal. Therefore, the universal energy equation needs to be modified. Assuming \( \frac{\rho dv}{g_e dl} \) in the universal energy equation is 0, the equation of motion including the Darcy-Weibath equation becomes

\[
\frac{dp}{dl} = g \frac{\rho}{g_e} \sin \beta + f \frac{\rho v^2}{2g_e d}
\]

This formula provides the liquid fluid pressure gradient in the declining pipe and the decreasing pressure magnitude in the range of pipe whose length is \( l \). If the units usually used in oil field analysis are applied, the following equation can be used

\[
\Delta p = \frac{1}{144} \rho l \sin \beta + 1.29 \times 10^3 \frac{fl\rho v^2}{d}
\]

in which \( \Delta p \), \( \rho \), \( \beta \), \( v \), and \( d \) are decreasing pressure magnitude, liquid density, declining angle of pipe, liquid velocity and pipe diameter, respectively.

**Turbulent model**

In order to simulate accurately fluid flow in a sucker rod pump, \( K - \varepsilon \) two-equation model is applied in the paper. It is assumed that crude oil is incompressible, forces other than pressure do not act on liquid, and the fluid state in the pump is steady. The following are several concrete equations.

The equation of continuity with constant density liquid is

\[
\nabla \cdot \mathbf{U} = \frac{\partial U_i}{\partial X_i} = 0
\]

The equation of momentum is

\[
\frac{\partial (U_i U_j)}{\partial X_j} = -\frac{\partial P}{\partial X_i} + \nu \frac{\partial}{\partial X_i} \left( \frac{\partial U_j}{\partial X_j} \right) \quad (i=1,2)
\]

in which \( P \), \( U \), and \( X \) are pressure, velocity and coordinate axis, respectively. But the \( i \) is the subscript for coordinate axis and it indicates the direction of coordinate axis.

Navier-Stokes equation is

\[
\frac{\partial \rho U_i}{\partial t} + \frac{\partial (\rho U_i U_j)}{\partial X_j} = B_i - \frac{\partial P}{\partial X_i} + \nu \frac{\partial}{\partial X_i} \left( \frac{\partial U_i}{\partial X_i} \right) + \mu \frac{\partial U_i}{\partial X_j} + \frac{\partial}{\partial X_i} \lambda \frac{\partial U_j}{\partial X_j}
\]
in which $B_i$ and $i$ are the mass force of unit volume and the subscript for coordinate axis.

Velocity can be divided into two parts such as mean velocity and fluctuant velocity. Let $P = \bar{P} + P'$, $U_j = \bar{U} + U_i''$, therefore, Navier-Stokes equation becomes

$$\frac{\partial (U_i U_j)}{\partial X_j} = -\frac{\partial P}{\partial X_i} + \nu \frac{\partial}{\partial X_i} \left( \frac{\partial U_i}{\partial X_j} + \frac{\partial U_j}{\partial X_i} \right) \quad (i=1, 2) \quad (9)$$

This equation is often called the mean Reynolds equation. Compared with the equation of momentum, the mean Reynolds equation has an additional part called Reynolds stress, which is a result of turbulent liquid fluctuation. Which turbulent model is obtained depends on how the additional stress is dealt with. With the help of the Boussinesq hypothesis, Reynolds stress can be connected with the mean strain ratio by imitating the relation of stress and strain when liquid flows in laminar flow manner. The related coefficient $\nu_t$, called the turbulent viscosity coefficient, is added to the mean Reynolds equation to make

$$\frac{\partial (U_i U_j)}{\partial X_j} = -\frac{\partial P}{\partial X_i} + (\nu + \nu_t) \frac{\partial}{\partial X_i} \left( \frac{\partial U_i}{\partial X_j} \right) \quad (i=1, 2) \quad (10)$$

in which $P$ is an effective pressure that is the sum of former pressure and extra pressure. The equation cannot be solved, so an additional equation should be referred to in order to calculate.

The equation of turbulent viscosity coefficient is

$$\nu_t = c_\mu \frac{K^2}{\varepsilon} \quad (11)$$

The equation of $K$ is

$$\frac{\partial (U_i K)}{\partial X_j} = \frac{\partial}{\partial X_j} \left( \frac{\partial K}{\partial X_j} \right) + \frac{\partial}{\partial X_i} \left( \frac{\partial U_i}{\partial X_j} \right) + \nu_t \frac{\partial}{\partial X_j} \left( \frac{\partial U_i}{\partial X_j} + \frac{\partial U_j}{\partial X_i} \right) - \varepsilon \quad (12)$$

The equation of $\varepsilon$ is

$$\frac{\partial (U_j \varepsilon)}{\partial X_j} = \frac{\partial}{\partial X_j} \left( \frac{\partial \varepsilon}{\partial X_j} \right) + c_\varepsilon \varepsilon \frac{\partial}{\partial X_i} \left( \frac{\partial U_i}{\partial X_j} + \frac{\partial U_j}{\partial X_i} \right) - c_2 \frac{\varepsilon^2}{K} \quad (13)$$

in which $l$ is the length of turbulent fluctuation range.

Turbulent fluctuating kinetic energy is calculated by

$$K = \frac{1}{2} \left( U''^2_1 + U''^2_2 \right)$$

Turbulent dissipation ratio is calculated by

$$\varepsilon = C_\varepsilon \frac{K^2}{l}$$

The magnitude of other experiential coefficient can be proposed as follows:
There are five unknown quantities, \( K \), \( P \), \( U \) (two numbers), and \( \varepsilon \) in the previous equation group, and there are also five equations, a continuity equation, a \( K \) equation, a \( \varepsilon \) equation, and two momentum equations. So the equation group is complete and it can be computed.

**Establishment of numerical analysis model**

Because the pump, piston, and valve are axisymmetric geometric models and the force with which the sucker rod pump is loaded also is axisymmetric, a two-dimensional model is applied to simulate the fluid flow in the pump. The model of the liquid path in a sucker rod pump is shown in Figure 1. The finite element model can be established according to the size of two types of sucker rod: 25-150RHACBMX-X, called pump 38, and 25-175RHACBMX-X, called pump 44 in China. The finite element model is shown in Figure 2 and Figure 3.

In this paper, according to the practical fluid flow of liquid and the boundary conditions, the boundary of the finite element model can be assumed as follows:

1. The velocity of the contact areas where liquid and solid contact each other is zero according the coupling character of multi-field;
2. The degree of freedom for pressure during exit of the pump is zero;
3. Reference pressure is standard atmospheric pressure;
4. The influence of temperature is not taken into account;
5. The variation of the whole sucker rod pump is minute.

\[
c_1 = 1.44, \quad c_2 = 1.92, \quad c_\mu = 0.09, \quad \sigma_k = 1.0, \quad \sigma_\varepsilon = 1.3
\]
Figure 2. The finite element upside model

Figure 3. The finite element downside model
Analysis of calculated results

In this study, the liquid in the sucker rod pump is crude oil whose density is $0.9 \times 10^3 \text{kg/m}^3$ and whose viscosity is 42cP. The temperature and atmospheric pressure that influence the pump and crude oil in this paper are normal temperature and standard atmospheric pressure, respectively.

Figure 4a. Liquid velocity vector (Entry of pump)

Figure 4b. Liquid velocity vector (Nether valve)
Figure 4c. Liquid velocity vector (Upper valve)

Figure 4d. Liquid velocity vector (The exit of pump)
Considering the sucking velocity of the pumping jack and character of the sucker rod pump, liquid velocity during entry of the pump in this study is between 0.5 and 5 m/s. In order to know more about turbulent fluid flow in a sucker pump, the liquid velocity during entry of the pump is put up to 5 m/s at first.

Liquid velocity vector and liquid streamline when liquid velocity in entry of pump is 5 m/s are shown in Figure 4 and Figure 5, respectively. It can be seen from Figure 4 and Figure 5 that there is obvious turbulent flow phenomenon in both valve positions and tortuous liquid path, which is in agreement with the observed experiment results. Figure 4 also shows that velocity is very large near the bottom of valve ball and the maximal liquid velocity appears at the bottom of nether moving valve ball, but the velocity near the top of the valve ball is negative and there is obvious refluxing. Moreover, the tortuous position in pump also has the obvious refluxing phenomenon and the velocity in local position is negative. Velocity contour lines are dense near the valve ball, which can be seen from Figure 4, so the loss of energy is more serious and velocity changes because of the tortuous liquid path. According to Bernoulli equation, the liquid pressure varies obviously with the change of liquid velocity. From the previous analysis, we can draw the conclusion that the valve is the key part that leads to most of energy loss in sucker rod pumps.

Figure 5a. The entry of pump
The change of velocity along pump is shown in Figure 6 (pump 38). It can be seen that there is obvious velocity fluctuation near the two valves and the maximal velocity difference is near to 2.6 m/s. The velocity during exit of the pump is 3.316 m/s and the velocity loss is almost 1.7 m/s. The change of liquid pressure along the pump is shown in Figure 7 (pump 38). It can be seen that there is obvious loss of pressure near the two valves and the maximal pressure loss is near to 0.710 MPa with the pressure in the entry of pump being 0.710 MPa and the pressure in the exit of pump being 0 MPa. Compared with the upper moving valve, the pressure loss in nether moving valley is more serious. Therefore, the two valves are the key parts that lead to most energy loss in sucker rod pumps.
The values used for liquid velocity in this study are 0.5, 1, 2, 3.5, and 5 m/s. The results obtained using the finite element method are presented in Table 1 and Table 2.

Figure 6. Change of velocity along pump

Figure 7. Change of pressure along pump
Table 1. finite element results (pump 38)

<table>
<thead>
<tr>
<th>Results</th>
<th>0.5</th>
<th>1</th>
<th>2</th>
<th>3.5</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximal radial velocity (m/s)</td>
<td>1.075</td>
<td>2.077</td>
<td>4.081</td>
<td>7.075</td>
<td>10.040</td>
</tr>
<tr>
<td>Maximal axial velocity (m/s)</td>
<td>2.313</td>
<td>4.719</td>
<td>9.640</td>
<td>17.084</td>
<td>24.529</td>
</tr>
<tr>
<td>Maximal fluid pressure (MPa)</td>
<td>0.022</td>
<td>0.055</td>
<td>0.157</td>
<td>0.397</td>
<td>0.763</td>
</tr>
</tbody>
</table>

Table 2. finite element results (pump 44)

<table>
<thead>
<tr>
<th>Results</th>
<th>0.5</th>
<th>1</th>
<th>2</th>
<th>3.5</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximal radial velocity (m/s)</td>
<td>0.797</td>
<td>1.646</td>
<td>3.352</td>
<td>5.775</td>
<td>7.969</td>
</tr>
<tr>
<td>Maximal axial velocity (m/s)</td>
<td>4.337</td>
<td>8.696</td>
<td>17.428</td>
<td>30.553</td>
<td>43.624</td>
</tr>
<tr>
<td>Maximal fluid pressure (MPa)</td>
<td>0.032</td>
<td>0.093</td>
<td>0.290</td>
<td>0.773</td>
<td>1.510</td>
</tr>
</tbody>
</table>

The effect of entry velocity of pump on maximal velocity in sucker rod pumps is shown in Table 1, Table 2, and Figure 8. It can be seen from Figure 8 that the relationship in both pump38 and pump44 between entry velocity and maximal fluid velocity is linear. The relationship between entry velocity of the pump and maximal pressure in sucker rod pump is shown in Figure 9. It can be seen that the relationship of both pump38 and pump44 between entry velocities and maximal fluid pressure is nonlinear. With the increasing of entry velocity, the liquid pressure also increases. Moreover, compared with pump 38, pressure loss of pump 44 is almost 1.7 times than pump 38 and the loss of velocity also is more than pump 38. But, from the velocity distribution in exit of the sucker rod pump, pump 44 has more advantages than pump 38.

Figure 8. Relation between entry velocity and maximal velocity of sucker rod pump
Result and discussion

Based the previous analysis, many conclusions can be drawn. The relationship between sucker rod pump entry velocity and maximal fluid velocity in pump is linear. However, the relationship between entry velocity and maximal fluid pressure is nonlinear. A pump with a larger diameter has a lower liquid pressure than a pump with a smaller diameter.

The method for using finite element analysis to analyze and study the velocity and liquid pressure distribution of a sucker rod pump is provided and proposed in this paper, and may offer some data for the engineering design of sucker rod pumps. The results indicate that this method can simulate the fluid in sucker rod pump reliably. The operation of a finite element analysis is convenient and requires less equipment than traditional method of experiment; the result of this method is identical to that of traditional method. Therefore it is a practical and valuable method for analyzing and studying the velocity and liquid pressure distribution of sucker rod pumps, and can also be used to study the fluid flow for other pump types.

References


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