Tapered Roller Thrust Bearing and Support Structure

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Abstract
Deflection of the support structure of a tapered roller thrust bearing moves the load inboard along the roller-to-raceway contact. The race and roller have tighter curvatures inboard, so moving the load inboard increases contact stress and decreases bearing life. Modifying the support structure moves the load outboard, reduces contact stress and increases life. The ANSYS finite element software modeled the shaft, bearing bottom plate, bearing top plate and lower support structure. The model also included the roller as a collection of spring elements, connected to a rigid region. This construct modeled load transmission by the roller as well as roller tilting, while avoiding time-consuming surface-to-surface contact calculations. Because all rollers bear the same load in a thrust bearing, a two-dimensional axisymmetric model sufficed to capture the behavior of the bearing. The program output included plots of stress and deflection in the bearing and in the support components. The output also included a listing of spring element forces, which characterized the load per unit length along the roller-to-raceway contact. Iterating over several modifications of the support structure allowed minimization of the maximum contact stress between roller and raceway, and so optimized bearing life.

Introduction
Rolling element bearings reduce friction to allow relative movement of two bodies in the presence of an applied load. Radial bearings allow the rotation of a shaft under a load applied radially, that is, transverse to the shaft axis. Thrust bearings allow the rotation of a shaft under a load applied axially, along the axis of the shaft. The rolling elements may consist of spherical balls in ball bearings or of rollers in roller bearings. In cylindrical roller bearings, the rollers consist of cylinders. In tapered roller bearings, the rollers consist of truncated cones.

Tapered roller thrust bearings carry axial loads and include conical rollers. Figure 1 shows the lower plate and several rollers from the bearing modeled in this study. Figure 2 more closely shows the tapered rollers. The fully assembled bearing would include enough rollers to fill up the raceway on the bottom plate, and would also include a top plate, similar to the bottom plate. The top and bottom plates contain the rollers between flanges. In applying this bearing, the bottom plate gets locked onto a shaft, while the top plate bears against a stationary structure. The bearing rollers transmit the axial load from the shaft to the structure, while allowing the shaft to rotate freely.

Figure 1. Tapered Roller Thrust Bearing
In the application modeled, the support structure consisted of the components shown in Figure 3. The bottom plate of the bearing rests on a backup plate. In turn, a lockring keeps the backup plate from sliding along the shaft. Because the lockring supports the backup plate only near its inside diameter, the backup plate can easily deflect downward near its rim, allowing the bottom plate of the bearing to also deflect downward near its outside diameter.
The load carried by each increment of length of the roller-to-plate contact strongly affects the compressive stresses engendered there. The curvatures of the plate and the roller also affect the stresses. The conical shapes of the plates and of the roller cause a decrease in the radius of curvature of the bodies as the radial coordinate decreases toward the axis of the bearing.

The deflection of the bottom plate near its rim moves the contact pressure between roller and bottom plate radially inward toward the axis of the bearing. Near the axis of the bearing, both roller and plates have smaller radii of curvature, so stresses increase.

Widely used texts on rolling element bearing analysis do not include detailed calculations for contact stresses in tapered roller bearings. One book (reference 1) suggests taking the average diameter of the tapered rollers as the diameter of the rollers in an equivalent cylindrical roller bearing.

More accurate analytical methods, such as that in reference 2, require the evaluation of complicated deflection functions, and the solution of a system of nonlinear equations by a custom-written computer program. The lack of easily implemented accurate stress calculation methods for tapered roller bearings motivated the present study.

This study considered modifying the backup plate to change the deflection of the backup plate and bottom plate to more favorably distribute the roller-to-plate contact pressure. Specifically, dishing the top surface of the backup plate might move the contact force radially outward. Dishing the backup plate refers to reducing its height linearly from a maximum at the OD to a minimum at the ID. The amount of dishing refers to the overall height difference between OD and ID. Figure 4 illustrates dishing.

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Figure 4. Dished Backup Plate

**Procedure**

An appendix to this paper "Validation of the Lamina Method and Comparison to a Three Dimensional Contact Model" establishes the accuracy of the lamina method and the difficulties with accuracy and long run times that characterize solid modeling of contact phenomena. A preliminary study of the effect of contact element stiffness on the deflection of the bearing bottom plate established the needed contact element stiffness to calculate accurate deflections. A second preliminary study of the effect of the number of spring elements, which modeled the roller, on the load distribution along the raceway established the number of spring elements needed for accurate roller load distribution. The main study of the effect of dishing the backup plate on the distribution of load along the raceway established the amount of dishing.
that minimized the maximum compressive stress in the roller-to-plate contact. Reducing the contact stress increases the life of the bearing.

**Analysis**

In all the models, PLANE42 elements modeled the bearing plates and support structure components. The modulus had magnitude 3E7 psi and Poisson’s ratio had magnitude 0.3, to simulate steel. Keyopt 3 for the element had value of 1, to set the axisymmetric option.

In an axisymmetric analysis, loads get applied on a 360-degree basis. The top plate of the bearing therefore bears the entire axial load. Under this scheme, ANSYS will correctly calculate deflections in the bearing plates and support structure.

**Preliminary Model 1: Evaluating Needed Contact Element Stiffness**

Deflections of the top and bottom plates of the bearing can strongly affect the transmission of load between the plates by the rollers. The plate deflections can change the location and the magnitude of the compressive stresses between roller and plate. For this reason, the model must calculate accurately the deflections of the plates.

A preliminary model of the bottom plate, backup plate, lockring, and shaft served to evaluate the needed contact element stiffness to accurately model deflections caused by load transmitted through the bottom bearing plate and support structure. The geometry of the components included the interference fits between shaft and plates. CONTAC48 elements with elastic Coulomb friction modeled the load transmission between the components. All runs employed friction coefficient of 0.3.

Over the course of five program executions, increasing the stiffness of the contact elements eventually produced deflection results with no significant change between runs. Figure 5 shows the vertical deflection plot for the fifth run of this model. ANSYS exaggerates the deflections according to the default display parameters. The figure also labels the components of the support structure and the bottom plate of the bearing.

![Figure 5. Preliminary Model 1 - Deflections](image)
Contact element stiffness varied from $3E5$ in the initial run to $3E9$ in the final run. The variation in contact stiffness strongly affected the penetration allowed at the contact elements, which in turn strongly affected maximum displacement, calculated at the OD of the bottom plate. Table 1 shows the convergence of the deflection with increasing stiffness. Contact element normal stiffness also strongly affected the number of iterations required for convergence. Subsequent runs of the full model employed the contact element stiffness $3E9$.

### Table 1. Deflection at OD of Bottom Plate vs Contact Element Normal Stiffness

<table>
<thead>
<tr>
<th>Run</th>
<th>Normal Stiffness Real Constant</th>
<th>Deflection, thousandths</th>
<th>Penetration, thousandths</th>
<th>Iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>$3E5$</td>
<td>166.27</td>
<td>79.40</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>$3E6$</td>
<td>unrecorded</td>
<td>10.70</td>
<td>9</td>
</tr>
<tr>
<td>5</td>
<td>$3E7$</td>
<td>15.70</td>
<td>2.00</td>
<td>14</td>
</tr>
<tr>
<td>6</td>
<td>$3E8$</td>
<td>7.94</td>
<td>0.30</td>
<td>22</td>
</tr>
<tr>
<td>7</td>
<td>$3E9$</td>
<td>6.25</td>
<td>0.03</td>
<td>69</td>
</tr>
</tbody>
</table>

**Preliminary Model 2: Evaluating Required Number of LINK10s**

A tapered roller contacts the bearing plate along a line. The curvatures of the tapered roller and of the plate change along the line of contact, and at any point along the line of contact the instantaneous values of curvature govern the load vs. deflection relation. Considering a short length of the tapered roller as a short cylindrical roller allows modeling the load vs. deflection relation. The term lamina in rolling element bearing technology refers to the short length, or slice of roller. The lamina method consists of modeling the tapered roller as an assembly of short cylindrical rollers, or laminae (reference 3). Figure 6 illustrates the concept.

![Figure 6. Lamina Method](image)

**The Lamina Method**

A cylindrical roller also contacts a plate along a line. The equation for Hertzian line contact describes deflection of these contacting bodies under load. The figure "Hertzian Line Contact vs Linear Spring" shows the load versus deflection relationship for the line contact (reference 1), and also shows the load vs. deflection relation for a linear elastic spring. The small departure of the line contact relation from the linear elastic relation suggests that a linear spring can model the behavior of a Hertzian line contact (reference 4). For a contact between bodies of given length, the slope of the line that most closely fits the load vs. deflection relation for the contact serves as the spring constant.
Figure 7. Hertzian Line Contact vs Linear Spring

Modeling each lamina in a tapered roller as a linear spring allows modeling the tapered roller as an assembly of springs. Figure 8 illustrates this concept.

Figure 8. Assembly of Linear Springs

However, application of the lamina method to a tapered roller requires further adaptations. The simple replacement of laminae with linear springs misrepresents the direction of the force exerted by the tapered roller on the plate. The contact force lies along the common normal line to the surfaces of the tapered roller and the bearing plate. The simple replacement of laminae with linear springs also does not allow rigid body rotation of the roller. Finally, simply replacing laminae with linear springs ignores the contacts between the roller ends and the flanges of the bearing plates.
Figure 9 shows a second preliminary model. Here BEAM3 elements model the body of the roller as a rigid region, while properly oriented LINK10 elements account for the elastic deflection at the roller-to-raceway contacts, and at the roller-to-flange contacts. The LINK10 elements include a gap option, so that they transmit only compressive forces. Multiplying the stiffness of the LINK10 spring elements by the number of rollers in the bearing accounted for the application of loads on a 360-degree basis in the axisymmetric model.

Figure 9. Preliminary Model 2 - Elements

Attaching LINK10 elements to BEAM3 elements on one end, and to PLANE42 elements at the other end, requires control of the mesh to place nodes in the proper orientation. Figure 10 more clearly shows the mesh. The LINK10 elements lie perpendicular to the surface of the bearing plate.
This second preliminary model examined the transmission of load from the top plate to the bottom plate across a roller. Figure 11 shows the distribution of compressive stress in the top and bottom plates. The LINK10 elements exert concentrated forces at nodes of the plates, so the calculated stress field does not accurately model the roller to raceway compressive stress. However, the force exerted by the LINK10 element, divided by the spacing between elements, does accurately model the load per unit length of contact exerted by the roller on the raceway. The appendix to this paper "Validation of the Lamina Method", establishes this point.
Figure 12 shows the force exerted by each LINK10 element divided by the spacing between elements. The curve in red shows the load per length results for a model with ten LINK10 elements along the roller length, and the curve in blue shows the load per length results for a model with twenty LINK10 elements. The similarity of the two curves suggests that twenty LINK10 elements capture the variation in load per length over the contact between roller and plate. Subsequent runs of the full model employed twenty LINK10 elements along the length of the roller.

As a later section will show, knowing the load per length and the roller and race geometries, the contact stresses can be calculated.

The bottom bearing plate in this preliminary model rested on rigid supports, specified by setting displacement $U_Y = 0$ for the nodes along the bottom of the bottom plate. This preliminary model therefore provided results for the rigidly supported bearing. Results for the rigidly supported case later get compared against results for the bearing on the actual support structure.

**Full Model: Evaluating Effect of Dishing on Contact Stress**

Figure 13 shows the axial normal stress in the bearing plates and support structure, where the backup plate has no dishing. The yellow areas near the roller-to-plate contacts show that the maximum contact loads lie inboard, nearer to the axis of the bearing than to the rim.
Figure 13. Unmodified Support Structure SY

Figure 14 shows the axial normal stress in the bearing plates and support structure, where the backup plate has 0.012 inch dishing. The yellow areas near the roller-to-plate contacts show that the maximum contact loads lie more evenly distributed than in the un-dished case.
Figure 14. 0.012 inch Dished Support Structure SY

Similar models included dishing amounts from zero to 0.015 inches of dishing, in 0.003-inch increments. Each model ran in about two minutes of CPU time, and required about 25 substeps and 50 total iterations, despite the nonlinear LINK10 and CONTAC48 elements.

Analysis Results & Discussion

Figure 15 compares the distribution of load along the roller-to-bottom-plate contact for the bearing in combination with three different support structures. The blue line, for the rigidly supported bearing, shows the most uniform distribution of load along the contact. The green line, for the bearing supported by an undished backup plate, shows load concentrated inboard, toward the axis of the bearing, in the region of tightest plate and roller curvatures. The red line, for the bearing supported by a backup plate having 0.012 inch dishing, shows load skewed slightly outboard, toward the rim of the bearing, in the region of milder plate and roller curvatures.
Calculation of the contact stresses and subsequent calculation of the stresses' effect on bearing life serves to evaluate the effect of the load distribution along the roller. Calculation of contact stress requires knowledge of the principal curvatures of the roller and of the bearing plate, each of which has a conical shape. Calculation of stress also requires the actual length for the contact.

Each LINK10 element simulates a lamina, or short length of roller, in contact with an equal length of bearing plate. The length of the lamina in contact depends on the slope of the conical surface of the bearing plate as in the equation:

\[ L_{\text{slope}} = \frac{L_{\text{axial}}}{\cos(\alpha)} \]

Where \( \alpha \) denotes the complement of one-half the included angle at the cone vertex, and \( L_{\text{axial}} \) denotes the length of the lamina, measured parallel to the axis of the roller.

Along the line of contact, the roller and raceway have zero curvature, or infinite radius of curvature. The other principal axis of curvature lies in the plane of transverse curvature, which plane lies perpendicular to the conical surface of the roller or bearing plate. Figure 16 shows a two-dimensional schematic of the conical surface, line of contact, and plane of transverse curvature.

Figure 15. Force vs. Radius

Figure 16. Plane of Transverse Curvature
Figure 17 shows a three-dimensional schematic of the conical surface and the plane of transverse curvature. The intersection of the plane of transverse curvature and the surface of the cone forms a parabola. The curvature of this parabola at its intersection with the line of contact governs the stress at the point of interest.

The equation

$$y^2 = k^2(x^2 + y^2)$$

defines the conical surface, where $k = \tan(\alpha)$. The equation

$$y - y_0 = -(1/k) \cdot (x - x_0)$$

defines the planar surface, where $(x_0, y_0)$ denotes the point where the surfaces intersect in the xy plane. The parameter $x_0$ also denotes the radial distance of the point of interest from the axis of the bearing.

Figure 18 shows a coordinate system in which the equation of the plane becomes $x = 0$. By translating the origin to $(x_0, y_0)$ and rotating through the angle $\alpha$, the equation for the cone becomes

$$(x \cdot \sin(\alpha) + y \cdot \cos(\alpha) + y_0)^2 = k^2 \cdot [(x \cdot \cos(\alpha) - y \cdot \sin(\alpha) + x_0)^2 + z^2].$$

Substituting $x = 0$ into the equation for the cone gives the equation for the curve of intersection of the cone and plane

$$(y \cdot \cos(\alpha) + y_0)^2 = k^2 \cdot [(x_0 - y \cdot \sin(\alpha))^2 + z^2].$$

Manipulating this last result produces an expression of the form
where \( c_3 \) and \( c_4 \) depend on \( \alpha \) and \( x_0 \).

\[
(y + c_3)^2 = c_4 * z^2 + c_3^2
\]

Figure 18. Rotated Coordinate System

To find the local radius of curvature requires evaluating the following expression at \( x = x_0, y = y_0 \):

\[
r = \left[1 + \left(\frac{dy}{dz}\right)^2\right]^{3/2}/\left(\frac{d^2y}{dz^2}\right).
\]

This evaluates to

\[
r_{race} = x_0 / \sin(\alpha),
\]

the local radius of curvature at the point \((x_0, y_0)\) of the conical bearing plate in the direction transverse to the line of contact. A similar analysis for the conical roller shows

\[
r_{roller} = y_0 / \cos(\alpha).
\]

As detailed in the preceding paragraphs, the lamina method implemented in the ANSYS finite element model calculates the distribution of load over the length of the roller. Having the load per unit length and the principal curvatures as a function of the location along the line of contact allows calculation of the contact stress.

The contact stress increases with the reciprocal of the curvature sum:

\[
\Sigma \rho = \left(1/r_{race}\right) + \left(1/r_{roller}\right).
\]

Substituting the curvature radii into the curvature sum:

\[
\Sigma \rho = [\sin(\alpha) / x_0] + [\cos(\alpha) / y_0].
\]

But

\[
y_0 = x_0 * \tan(\alpha),
\]
The maximum contact stress in a line contact, 

$$\sigma = \sqrt{(F \cdot \Sigma \rho) / (\pi \cdot L \cdot C)} ,$$

where $F$ denotes load, $L$ denotes length of the contact, and $C$ includes material elastic properties (reference 1). Substituting for the curvature ratio and for the length of the lamina in contact,

$$\sigma = \sqrt{F / \left[ \pi \cdot x_0 \cdot \sin(\alpha) \cdot L_{\text{slope}} \cdot C \right]} .$$

This equation shows that, all else constant, the contact stress will decrease along the conical roller/plate contact as $1/\sqrt{x_0}$ . Recalling that $x_0$ measures radial distance from the bearing axis, a bearing that distributes load radially outboard will have lower maximum contact stress.

Figure 19 shows $\sigma$ versus $x_0$, i.e. the distribution of contact stress along the length of the contact between the roller and the bearing bottom plate. The figure shows three curves, for the bearing in combination with three different support structures.

![Figure 19. Contact Stress versus Radius](image)

The bearing having zero dishing develops the highest contact stress. The rigidly supported bearing, with the most evenly distributed load, nonetheless develops higher maximum stress than the bearing with backup plate dished by 0.012 inches. While the rigidly supported bearing distributes load most evenly along the contact, the 0.012 inch dished bearing distributes load to outboard regions of low roller and plate curvature, and therefore distributes stress most evenly, and develops the lowest maximum stress.

Stress calculations for a cylindrical roller thrust bearing having curvatures roughly equivalent to the tapered roller thrust bearing served to check the order of magnitude of the calculated stresses for the tapered roller bearing. The rollers of the cylindrical roller bearing had curvature equal to the average curvature of the tapered rollers. Because the curvature sum $\Sigma \rho$ does not change with location along the contact in a cylindrical roller bearing, the relation $\sigma = \sqrt{(F \cdot \Sigma \rho) / (\pi \cdot L \cdot C)}$ serves for calculation of the
maximum contact stress in the cylindrical roller bearing. The table "Contact Pressures and Lives" reports
the stress for this approximately equivalent cylindrical roller bearing.

The life of a roller bearing depends with great sensitivity on the contact stress in the bearing, as shown by
the proportionality $L \propto \sigma^8$ (reference 1). To find the ratio of the life of a bearing to the life of a
benchmark bearing then take the inverse ratio of the contact stresses in the bearings to the eighth power.
Taking the rigidly supported bearing as the benchmark case, Table 2 shows the relative lives of the bearing
as installed on support structures having various dishing of the backup plate.

<table>
<thead>
<tr>
<th>Dishing, inches</th>
<th>Max Contact Pressure, ksi</th>
<th>Relative Life</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cylindrical</td>
<td>145</td>
<td></td>
</tr>
<tr>
<td>Rigid Support</td>
<td>159</td>
<td>1</td>
</tr>
<tr>
<td>0.000</td>
<td>187</td>
<td>0.27</td>
</tr>
<tr>
<td>0.003</td>
<td>177</td>
<td>0.42</td>
</tr>
<tr>
<td>0.006</td>
<td>166</td>
<td>0.71</td>
</tr>
<tr>
<td>0.009</td>
<td>157</td>
<td>1.11</td>
</tr>
<tr>
<td>0.012</td>
<td>148</td>
<td>1.77</td>
</tr>
<tr>
<td>0.015</td>
<td>151</td>
<td>1.51</td>
</tr>
</tbody>
</table>

The stresses for the tapered roller thrust bearing cases all have the same order of magnitude as the stress for
the cylindrical roller thrust bearing of roughly equivalent dimension.

The bearing on the support structure with 0.000 inch dishing attains only 27 per cent of the benchmark life,
while the bearing on the support structure with 0.012 inch dishing attains 177 percent of the benchmark
life. In other words, careful selection of the amount of dishing increases the bearing life by 6.5 times over
the case of zero dishing.

**Conclusions**

The lamina method, as implemented by spring elements in a finite element model of a crowned cylindrical
roller, provides contact distribution results within 1.7% of theoretical values.

A three-dimensional, quarter-symmetry roller to raceway contact model, despite efforts to refine mesh
adequately to capture the very narrow contact width, overestimated maximum contact pressure by 17%.

The three-dimensional contact model, despite substructuring, required very long run times compared to the
lamina method.

The stiffness of the contact elements in the support structure model strongly affects the calculated
deflections. Parametric study of this effect allowed selection of sufficient stiffness to achieve accuracy.

The number of spring elements in a model of roller-to-plate contact according to the lamina method exerts
a moderate effect on the calculated load distribution in the contact. The rollers in this study did not have
sharp discontinuities in their surface profiles.

The lamina method, as implemented by properly oriented spring elements in a finite element model,
provides useful load distribution results for tapered roller bearings.
Deflection of the support structure for a tapered roller bearing strongly affects the distribution of load over the roller-to-plate contact, the maximum contact stress, and the life of the bearing.

A controlled amount of dishing, a modification of the backup plate supporting the bearing bottom plate, can compensate for deflection of the support structure to optimally distribute load over the roller-to-plate contact to minimize contact stress and maximize bearing life.

**Appendix: Validation of the Lamina Method and Comparison to a Three Dimensional Contact Model**

The validation described here, against analytical results for a crowned cylindrical roller, employs standard Hertzian contact calculations (reference 1) as well as the CPATCH computer program written at Emerson Power Transmission (reference 5).

The comparison model described here, against the same roller and raceway geometry, employs solid elements and CONTAC49 surface-to-surface elements.

Figure 20 shows the geometry of the crowned cylindrical roller. The roller resembles a barrel in shape.

![Figure 20. Roller Geometry](image)

**Lamina Model of Crowned Cylindrical Roller**

The figure "Lamina Geometry" shows the division of the roller into several slices, or laminae, and the modeling of each lamina by a LINK10 compressive spring element.
The model accounts for the roller crown by the initial strain real constant for the LINK10 element. The initial strain in the element (ISTRN) is given by $\Delta/L$, where $\Delta$ is the difference between the element length, $L$, (as defined by the I and J node locations) and the zero strain length, $L_0$. For the gap option, a positive strain indicates a gap condition. All the springs have nodes spaced $2*r_{Roller}$ apart, so that element length $L = 2*r_{Roller}$. However, the elements toward the end of the roller have lesser zero strain length $L_0$, so that $L-L_0$ equals the reduction in diameter caused by the crown.

The lamina method assumes that each short length of the roller behaves as a short Hertzian line contact (reference 3). Analytical solutions exist for the load versus deflection and stress versus load relationships for such a contact (reference 1). A linear spring models the load vs deflection behavior of each lamina, where the spring stiffness gets calculated as discussed in the section of this paper on Preliminary Model 2.

The figure "Lamina Forces" shows the finite element model, which consists entirely of two rows of nodes with the LINK10 spring elements connecting them. The nodes of the lower row get locked in place. The nodes of the upper row get coupled in their vertical displacements, and so behave as if on a rigid line.
The spring forces exerted by each LINK10 element color the element according to the key. Springs near the center of the contact carry more force, while springs toward the end of the contact carry less force, because of the crown. A listing of the force in each spring element constitutes the output of the simulation.

The maximum contact stress at each lamina,

\[ \sigma = \sqrt{\frac{F \sum \rho}{\pi L C}} \]

where \( F \) denotes the spring element force, \( L \) denotes length of the lamina, or spacing between the spring elements, and \( C \) includes material elastic properties (reference 1). The contact stress increases with the reciprocal of the curvature sum:

\[ \sum \rho = \left( -\frac{1}{r_{\text{race}}} \right) + \left( \frac{1}{r_{\text{roller}}} \right) + \left( \frac{1}{r_{\text{crown}}} \right) \]

where the negative sign denotes a concave, or outer race. From the forces in each spring element, this relation for \( \sigma \) computes the distribution of contact stress along the center of the roller-to-raceway contact.

For this study \( r_{\text{race}} = 2.015 \) inches, \( r_{\text{roller}} = 0.15745 \) inches, and \( r_{\text{crown}} = 26.3 \) inches.
**Hertzian Calculation of Stress for Crowned Cylindrical Roller**

The crowned cylindrical roller contacting an outer raceway constitutes a Hertzian point contact, for which reference 1 gives algorithms for contact pressure calculations.

**CPATCH Calculation of Stress for Crowned Cylindrical Roller**

The proprietary computer program CPATCH (reference 5), provides an additional check on the contact pressure distribution. The figure "CPATCH Contours" shows the contact pressure distribution that CPATCH calculated.

**Three Dimensional Model of Crowned Cylindrical Roller**

Figure 23 shows a quarter-symmetry model of the crowned cylindrical roller in contact with an outer race. Two orthogonal planes of symmetry pass through the center of the contact. SOLID45 elements modeled roller and race, and CONTAC49 elements established contact between roller and race.

![Figure 23. 3D Model](image)

Figure 24 shows the details of the mesh near the contact. Figure 25 shows the details of the mesh in the contact region on the outer race. The elements in the contact region serve to resolve the anticipated contact width of about 0.0035 inches. The radial and circumferential dimensions of the elements in the contact region therefore amount to about 0.00035 inches. The aspect ratio limitation imposed by ANSYS limits the
maximum axial dimension of the elements to about 0.007 inches, but the need to resolve the anticipated contact length of 0.086 inches reduces this to perhaps 0.002 inches. As the elements expand away from the contact region, the limit on the axial dimension and the aspect ratio limitation limits the circumferential and radial dimensions of the elements to about 0.040 inches. The quarter-symmetry model of the 0.3149 inch diameter roller and the outer race segment includes 57,321 elements, 59,638 nodes and 172,076 degrees of freedom. ANSYS estimated 13.3 minutes for each iteration of the solution.
Substructuring reduces CPU time for a nonlinear analysis. Substructuring the linear portion of the model obviates recalculating the element matrices for that portion at every equilibrium iteration (reference 6). Generation passes defined a superelement for the roller and a superelement for the outer. Only the contact elements and the master degrees of freedom for the superelements participated in the solution. ANSYS converged in 89 iterations and required 1.49 hours of CPU time. Without substructuring, the estimated time for 89 iterations amounts to 19.7 hours. An expansion pass for the outer superelement provided stresses in that portion of the model.

Figure 26 shows the distribution of radial compressive stress, or contact pressure, on the contact surface of the outer race. As expected from Hertzian theory, the contact region occupies one quarter of an ellipse, and the highest pressures tend to lie along the lower edge and left edge of the plot, along the centerlines of the contact ellipse. The axial centerline of the contact lies parallel to the axis of the roller and lies along the lower edge of the plot. The transverse centerline of the contact lies along the left edge of the plot.

Figure 26. 3D Model Contact Pressure

Figure 27 shows a closer view of the contact pressure distribution, near the region of maximum contact pressure at the center of the contact. Two irregularities appear, one at the extreme left edge, and a second farther to the right along the lower edge. The first irregularity appears as a region of high pressure at the extreme left edge of the plot. The pressure spike lies off the centerline of the contact, contrary to Hertzian theory. The second irregularity consists of a drop, then increase in the pressure as one traces along the lower edge, or axial centerline, of the contact from left to right. Hertzian theory calls for a monotonic decrease in contact pressure as one moves away from the center of a point contact.

Figure 27. 3D Model Contact Pressure Near Contact Center

Results of the Validation and Comparison

Figure 28 shows the variation of contact pressure along the axial centerline of the crowned roller's contact with an outer race. The maximum pressure occurs in the geometric center of the elliptical contact.
As discussed in the "Analysis Results and Discussion" section, the rolling contact fatigue life of the roller varies inversely as the eighth power of this contact stress. Table 3 shows pressures calculated by the four methods and the relative rolling contact fatigue life of the central portion of the roller.

Table 3. Crowned Roller Contact Pressures and Lives

<table>
<thead>
<tr>
<th>Calculation Method</th>
<th>Max Contact Pressure, ksi</th>
<th>Relative Pressure</th>
<th>Relative Life</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hertzian</td>
<td>175.5</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>CPATCH</td>
<td>176.2</td>
<td>1.004</td>
<td>0.967</td>
</tr>
<tr>
<td>Lamina</td>
<td>172.5</td>
<td>0.983</td>
<td>1.145</td>
</tr>
<tr>
<td>CONTAC49</td>
<td>205.3</td>
<td>1.170</td>
<td>0.285</td>
</tr>
</tbody>
</table>

The CPATCH program calculates pressure nearly identical to the Hertzian result.

The lamina method calculates maximum pressures within less than 2% of the Hertzian result. Subsequent life calculations that employ this pressure will lie within 15% of the Hertzian result.

The three-dimensional solid model overestimates maximum contact pressure by 17%. Calculations of life based on this pressure would underestimate life by 72%.

References

1. Harris, T.A. - Rolling Bearing Analysis, 3rd edition

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