Simulation of Rock Cutting by the Finite Element Method
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Abstract
The two dimensional simulation models of rock fragmentation due to cutting have been developed by the finite element method (FEM) using ANSYS software, in the models the friction between the crack faces and the tool with rock is taken into account. A special crack tip element and the surface-to-surface contact elements are used to simulate the feature of the crack tip stress singularity. The crack tip stress and displacement distributions can be accurately determined, and the stress intensity factor (SIF) and strain energy release rate and strain energy density factor can be calculated accordingly. Thus the crack initiation and propagation paths and required forces can be accurately determined by the rigorous fracture criteria (σ-criteria, G-criteria and S-criteria).

Several studies have been made to simulate rock-cutting phenomena by fracture mechanics principles coupled with the FEM. The cutting process of major chip formation by a drag pick cutting has simulated. The two-crack model simulation is performed for more attempts; as a result the influence of cutting parameter and tool geometry and friction and multi-crack in cutting process can be investigated by simulation.

Introduction
After a large number of tests a number of models describing drag bit cutting of rock have been presented by the empirical and analytical methods during the last decades, the perhaps two most commonly used models in these models were presented by Evans (1962) and Nishimatsu (1972). These are largely based on experimental observation. In both these models different failure criterions are used moreover different chip geometry is assumed and proposed. Thus I wonder what scope they apply to and how their validity is. We haven’t been able to understand both these models completely. Numerical methods such as the FEM must give a better result.

The stress state and crack propagation behavior involved in the rock cutting process is complex and have not been extensively researched. There is a need for improved understanding of the stress state and crack propagation behavior due to cutting in order to design desirable cutting tools and to achieve higher rock excavating efficiency, lower energy consumption and larger pieces of which this aspect is especially important for coal cutting due to high price.

The fracture mechanics approach incorporated with the finite element model allows the study of the complex stress state due to cutting, to accurately determine the strain energy release rate and the crack tip stress intensity factors (SIF). Therefore the crack initiation and propagation paths and required forces can be accordingly accurately determined by the rigorous fracture criteria, and the mechanism of chip formation can be better understood. In applying this approach a few assumption must be made: firstly, the crack tip fracture process zone (FPZ) is assumed to be so small that it can be ignored; secondly, one of the three fracture criteria (σ- criteria, G- criteria and S- criteria) is assumed to govern crack ignition and propagation, here σ- criteria is the maximum stress criteria; G- criteria and S- criteria are the maximum strain energy release and the minimum strain energy density respectively; next the rock is assumed to be homogeneous.

Simplified two dimensional (2-d) models have been developed by the FEM in order to simulate rock fragmentation and chip formation due to the parallel cutting, in the models the friction between the crack faces and the tool with rock is taken into account. A special crack tip element (isoparametric quadratic singular element) and the surface-to-surface contact elements are used to simulate the feature of the crack tip stress singularity. The strain energy release rate is calculated from the computed stress intensity factors which are obtained from the nodal displacements of the singular elements by the displacement correlation technique (DCT) and ANSYS displacement interpolation technique (DIT). The fracture criteria employed is mainly σ- criteria and G- criteria. The crack initiation and subsequent crack propagation is simulated in the process of major chip formation by incorporating an automatic remeshing process into parametric program - SRCI_2d and SRCP_2d. SRCI_2d is the program of the determination.
of the crack initiation orientation, and SRCP_2d is the program of the determination of the complete fracture propagation path. Figure 1 is the flowchart of the computer programme showing the procedures for crack propagation. A simplified computing model for the analysis of the rock cutting process is shown in Figure 2. The assumed dimension scales are shown in the diagram. H is the cutting depth; h is the contact height of cutting tool with rock, and p is the pressure distribution applying the contact height of rock body, α is the rake angle, cutting angle δ is equal to 90°-α; φ is the crack propagation direction. And ak is the crack length; DEFG is the boundary with symmetry restraints which can be lengthened. In the computing model the rock body is assumed linear elastic, elastic modulus E=1000MPa, Poisson’s ratio ν =0.3, cutting depth H=10-100mm, cutting angle δ=45° and 75°, the load applying height h_l= 0.1-0.2H.

Figure 1. Computer program showing the procedures for crack propagation
SIF and the crack propagation direction

The simulation of the rock cutting process due to a drag pick cutting aimed to compute the cutting load and the major chip formation, the simulation processes are to determine the crack initiation and propagation path. In order to calculate the crack initiation and propagation path by the FEM, the calculation of the stress intensity factors (SIF) and crack propagation direction are a key problem. The mixed mode SIFs are commonly computed using the displacement correlation technique (DCT). The crack tip element is to use the special isoparametric quadratic singular element. The computational error of DCT using the singular element has been studied in reference 1, and the SIFs can be accurately calculated by the adaptive meshing method [1][2]. With the SIFs the crack propagation direction is determined easily by the fracture criteria.

The comparison of the displacement correlation and ANSYS interpolation technique

The DCT is one of the simplest methods to evaluate SIFs. It consists of correlating numerical results for displacement at specific locations on the crack with available analytical solutions. When we employ the singular elements in FEA, for quarter point singular elements, the crack opening displacement (COD) and the crack sliding displacement (CSD) at \((x, y) = (-r, \pi)\) are given by [1]

\[
\begin{align*}
\left\{ \begin{array}{l}
COD(-r, \pi) \\
CSD(-r, \pi)
\end{array} \right\} &= \frac{r}{a_L} \begin{bmatrix}
(4U_1^* - U_2^*) \\
(4V_1^* - V_2^*)
\end{bmatrix} \\
\end{align*}
\]  

(1)

On the other hand, the analytical expression for COD (-r) and CSD (-r), neglecting higher order term, can be written as

\[
\begin{align*}
\left\{ \begin{array}{l}
COD(-r, \pi) \\
CSD(-r, \pi)
\end{array} \right\} &= \frac{k + 1}{\mu} \sqrt{\frac{r}{2\pi}} \begin{bmatrix}
K_I \\
K_{II}
\end{bmatrix} \\
\end{align*}
\]  

(2)

Therefore the SIFs for mode I and II can be evaluated by

\[
\begin{align*}
\left\{ \begin{array}{l}
K_{IDT} \\
K_{II,DCT}
\end{array} \right\} &= \mu' \begin{bmatrix}
4U_1^* - U_2^* \\
4V_1^* - V_2^*
\end{bmatrix}
\end{align*}
\]  

(3)

Here

\[
\mu' = \sqrt{\frac{2\pi}{a_L}} \frac{\mu}{1 + \kappa}
\]
Where material properties $\mu$ is shearing modulus, $k = 3 - 4\nu$ for plane strain, $k = (3 - \nu) / (1 + \nu)$ for plane stress, and $\nu$ denotes the Poisson’s ratio. $a_i$ is a characteristic length (singular element length) associated to the crack tip elements. $U^*$ and $V^*$ are the relative displacements in perpendicular and parallel to crack surface, respectively. 1 and 2 represent locations at quarter-point and end-point on crack tip surface.

ANSYS software uses the displacement interpolation technique (DIT)\(^4\) to compute the SIFs. DIT is dissimilar to DCT. Its accuracy is seemingly a little better than DCT, but it cannot give the sign of the SIF $K_{II}$ if it is negative.

**The crack initiation angle and propagation direction**

The crack initiation takes place at the direction with maximum tangential stress by $\sigma$- criteria, maximum energy release rate by $G$- criteria and minimum strain energy density by $S$- criteria. When a crack is exposed to a mixed model I-II loading, consider a polar- coordinate system ($r$, $\theta$), the crack tip stress can be written as follow:

\[
\begin{bmatrix}
\sigma_r \\
\sigma_\theta \\
\tau_{r\theta}
\end{bmatrix} = \frac{K_I}{\sqrt{2\pi r}} \begin{bmatrix}
1 + \sin^2 \frac{\theta}{2} \\
\cos^2 \frac{\theta}{2} \\
\sin \frac{\theta}{2} \cos \frac{\theta}{2}
\end{bmatrix} + \frac{K_{II}}{\sqrt{2\pi r}} \begin{bmatrix}
\sin \frac{\theta}{2} (1 - 3\sin^2 \frac{\theta}{2}) \\
-3\sin \frac{\theta}{2} \cos^2 \frac{\theta}{2} \\
\cos \frac{\theta}{2} (1 - 3\sin^2 \frac{\theta}{2})
\end{bmatrix}
\] (4)

According to $\sigma$- criteria the crack initial extension angle is equal to\[^3\]:

\[
\theta_m = 2 \arctan \left\{ \frac{1}{4} \left( \frac{K_I}{K_{II}} \pm \sqrt{\frac{K_I}{K_{II}} + 8} \right) \right\}
\] (5)

This is the formula presented by Erdogan and Shi (1963). $\theta_m$ is an orientation with respect to the crack original plane. The sign of the right equation (3) is determined by two order derivative of tangential stress $\sigma_\theta$ at the orientation $\theta_m$, that is

\[
\frac{\partial^2 \sigma_\theta}{\partial \theta^2} \bigg|_{\theta=\theta_m} < 0
\] (6)

Therefore the crack propagation direction is accurately calculated by SIFs. Certainly in the finite element model the crack initial extension angle can be computed by getting the location of maximum tangential stress in crack tip vicinity when SIFs cannot be obtained, such as complicated cracks such as open V-type cracks, but the error may be much great.

However the expression of energy release rate $G$ in the direction of $\theta$ is complex, the SIFs at the tip of a extended and branch crack can be computed directly by FEM. Thus the energy release rate $G$ should be equal to:

\[
G = \frac{1 - \nu^2}{E} \left( K_I^2(\theta) + K_{II}^2(\theta) \right)
\] (7)

\[
K_{\epsilon}(\theta) = \sqrt{K_I^2(\theta) + K_{II}^2(\theta)}
\] (8)

Thus the direction $\theta_m$ of maximum energy release rate can be found as a result of $G$- criteria, that is the direction of maximum equivalent SIF $K_{em}$.

In the other hand the general expression for the strain energy density $S$ can be expressed as:

\[
S = \frac{1}{2\mu} \left[ k + 1 \frac{1}{8} (\sigma_r + \sigma_\theta)^2 - \sigma_r \sigma_\theta + \tau_{r\theta}^2 \right]
\] (9)

Thus the direction $\theta_m$ by $S$- criteria can be calculated easily by means of the finite element post processing.
Table 1 is the result of the crack initiation angle of the different cutting angle. Figures 3-7 is the tangential stress distribution of the initial crack tip with respect to 45° and 75° cutting angle without and with friction. Figure 8 is the variation of SIFs with the crack direction angle. When cutting angle $\delta = 75^\circ$, there are two probable directions of maximum energy release rate, a tensile stress direction and other shear stress direction which is like Nishimatsu model. All results are calculated by APDL program SRC1_2d.

Table 1. Crack initiation angle $\phi$

<table>
<thead>
<tr>
<th>Cutting angle</th>
<th>45°</th>
<th>75°</th>
</tr>
</thead>
<tbody>
<tr>
<td>friction</td>
<td>0</td>
<td>0.2</td>
</tr>
<tr>
<td>$\sigma$- criteria</td>
<td>22.5°</td>
<td>42.5°</td>
</tr>
<tr>
<td>$G$- criteria</td>
<td>11°(16.5°)</td>
<td>29°</td>
</tr>
<tr>
<td>$S$- criteria</td>
<td>11°</td>
<td>29°</td>
</tr>
</tbody>
</table>

Figure 3. Crack tip tangential stress ($\sigma_y$) distribution of 45° rake angle

Figure 4. Crack tip tangential stress ($\sigma_y$) distribution of 45° rake angle with friction
Figure 5. Crack tip tangential stress ($\sigma_y$) distribution of 15° rake angle

Figure 6. Crack tip tangential stress ($\sigma_y$) distribution of 15° rake angle with friction

Figure 7. Crack tip tangential stress ($t_{xy}$) distribution of 15° rake angle with friction
Cutting process simulation

The program SRCP_2d has been used to simulate the rock cutting process due to drag picks. The $\sigma$-criteria and $G$-criteria are adopted in the model program. The complete fracture propagation path can be calculated, and the cutting force can also be calculated. It is apparent that the major chip formation caused by fracture propagation is similar to that proposed Evans [3]. In this model fracture toughness $K_{IC} = 0.31 + 0.027E^3$, Friction coefficient between tool and rock $\mu = 0.2$.

Crack propagation simulation of 45\textdegree cutting angle

For the cutting angle $\delta = 45^\circ$, the crack initiation angle is 11$^\circ$ and 29$^\circ$ respectively by $G$-criteria in table 1. The complete fracture propagation path is in Figures 9 and 10, respectively. The cutting chip with friction is a bit greater than one without friction. The cutting force is greater also.

Figure 8. That variation of SIF with crack direction angle

Figure 9. The complete fracture propagation path of 45\textdegree cutting angle
Figure 10. The compute fracture propagation path of 45° cutting angle with friction

Crack propagation simulation of 75° cutting angle

For the cutting angle $\delta=75^\circ$ the crack initiation angle is 54° and 69° respectively in table 1. The complete fracture propagation path is in Figures 11 and 12, respectively. The cutting chip with friction is very greater than one without friction, and the cutting chip is also much greater than one of the 45° cutting angle. The cutting fore is greater than one without friction and it of the 45° cutting angle. If the cutting angle $\delta$ is greater than 75°, the calculated cutting chip will be much great. This doesn’t resemble the actual cutting of rock. The other crack initiation angle -1° is good for the probable direction of crack propagation; the propagation path is similar to that proposed by Nishimatsu.

Figure 11. The complete fracture propagation path of 75° cutting angle
Cutting load

The cutting force is directly proportional to the fracture strength, cutting depth and the cutting width. It is consistent with the computational results of the finite element method. Using FEM the effects of the cutting angle and friction on the cutting force can be computed, and that the effects of multi-cracks on it can be modeled and studied.

Cutting load of double crack

In practice the mechanism of rock fragmentation by cutting is in general a complex process. In the vicinity of cutting tool tip it is possible to create several cracks. Such as when cutting angle $\delta=75^\circ$, according to $G$-criterion the crack initial extension direction have two angle likely. If there are two initial cracks, the influence on the cutting force must be produced. The computational result by using FEM shows the SIF is little greater than single crack. That is the cutting force will be greater. Thus the influence of multi-cracks on the cutting force can be understood. The stress of two initial cracks is shown in Figure 13.
Cutting load with different cutting depth and cutting angle

We are interested in the relation between the cutting force and the cutting depth and cutting angle and the influence of friction on the cutting force from the results of FEM calculation. It is depicted in Figure 14. It is apparent that the cutting force increases with increasing the cutting angle and the cutting depth and friction coefficient, and it increases with load contact height also.

![Figure 14. The cutting load with the cutting angle and cutting depth](image)

**Conclusion**

Using ANSYS software SIF can be computed accurately, and by APDL automatic program the simulation of rock cutting process can be well finished in the 2-d model. The cutting force and the major chip formation can be determined, and the influence of friction and multi-cracks on the cutting force can also be calculated. The relation of the cutting force with the cutting depth and the cutting angle can be obtained. The empirical and analytical models by Evans and Nishimatsu have been proved. The cutting force can be computed with the fracture toughness $K_{IC}$.

In order to better understand the mechanism of breakage process and chip formation and accurately determine the cutting force in rock cutting process, the three dimensional model and new FEM is required. By new computers quite real rock cutting process will be simulated.

**References**


