FEM Analysis of a CVT Pulley

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Abstract

A failure analysis has been carried out on a riveted half pulley mounted on the CVT system of the new Piaggio 850cc twin engine.

First of all, different design variants have been analysed with simplified models, in order to single out the one with the best performance.

The best configuration has then been re-analysed with a more detailed model, including the stress state induced by the riveting process, which has been simulated with a separate model. The working conditions corresponding to Piaggio standard endurance cycle have been used to estimate the fatigue damage withstood by the component.

Finally, the critical working condition has been singled out and the corresponding stress state has been compared with the one arising during Piaggio standard bench test for the component under examination.

Introduction

This paper is about the verification of the structural strength of a component of the new Piaggio 850 cc TWIN engine, which will equip a big Piaggio scooter. The engine is shown in the following figure.

![Figure 1. The Piaggio TWIN engine](image)

The component in question belongs to the continuously variable transmission (CVT) system, shown in the following figure.
Figure 2. The CVT system

By means of an asynchronous belt, the CVT system transmits the motion from the crankshaft to a rear transmission, which in turn transmits the motion to the rear wheel of the vehicle. The driven pulley, highlighted in the last figure, is made of 2 halves, as shown in the following figure:

Figure 3. The driven pulley
The component whose structural strength was to be verified is the fixed-half pulley. The reason why the verification had to be done was some failures happened during engine bench tests. The goal was to show that the design was sound and so that the causes of the failures were to be sought among other factors, such as the quality of the material, manufacturing process, etc.

**Procedure**

The half-pulley is fixed to the driven shaft by a riveted flange, as shown in the following figure:

![Figure 4. The riveted flange](image)
The failures happened near the rivets, as shown in the following figure:

![Figure 5. The critical zones](image)

The stress field in the half-pulley near the rivets has been computed, taking into account the contribution due to the riveting process. A simplified approach has been used to model both the riveting process and the combination of the stress state due to working conditions and to the riveting process. A simplified approach has been needed because of time and hardware constraints.

Besides that analysis, the predicted stress field under the working conditions has been compared with the one induced in the half-pulley during the standard Piaggio fatigue bench test for that component.

**Analysis**

The three standard Piaggio design conditions have been taken into account: maximum torque, maximum power and overspeed, the latter representing the engine running at a speed slightly larger than the maximum one. The first analysis step has consisted of a simplified comparison between different configurations proposed by the designers.

Once the most promising configuration had been singled out, its fatigue endurance has been predicted, combining the residual stresses due to the riveting process with the stress field computed with a detailed model simulating the working conditions.

Finally, the critical design condition has been singled out and the corresponding stress state has been compared with the one arising during Piaggio standard bench test for the component under examination.

**Simplified comparison**

This kind of analysis has been carried out assuming that the rivets exchange loads with the other components only through the following surfaces:
Figure 6. The load transferring surfaces
Physical continuity has been assumed between the rivets and the other components, through the surfaces shown above.
Only the portion of the flange has been considered, that is shown in the following figure:

Figure 7. The portion of the flange considered for the simplified comparison
An angular velocity field has been applied to the model, to simulate the engine speed.
It has been assumed that the belt exerts equal loads on each of the two half-pulleys. These loads have been assumed to be uniformly distributed over the belt/pulley contact surfaces.

The flange has been assumed to be fully constrained at the surface shown in the following figure:

![Figure 8. The constrained surface](image)

**Fatigue analysis**

The riveting process

Due to the strong nonlinearities involved in the riveting process, a simplified model has been used. A radial cross section of the pulley has been taken into account. The section is shown in the following figure, together with a rivet in its undeformed shape.
Two FEM models have been built, one for each of the parts of the cross section shown in the following figure:
Two axisymmetric solid have been considered, whose axial cross sections are the two parts shown above. Each of them has been analyzed under the load used to shape the rivet. The load has been applied by a moving punch, simulated as a rigid body. The two FEM models are shown in the following figure, which also contains the support that holds the rivet in position.

![Figure 11. Simulation of the riveting process](image)

The following coordinate system has been defined:

![Figure 12. Coordinate system for the simulation of the riveting process](image)

Let \( \mathbf{u}_1 \) and \( \mathbf{u}_2 \) be the displacement vector computed for the generic point of the axisymmetric models, for part #1 and #2. With reference to the last figure, \( \mathbf{u}_1 \neq \mathbf{u}_1 (\theta) \) and \( \mathbf{u}_2 \neq \mathbf{u}_2 (\theta) \), that is, \( \mathbf{u}_1 = \mathbf{u}_1 (r, z) \) and \( \mathbf{u}_2 = \mathbf{u}_2 (r, z) \). Let \( \mathbf{u} \) be the displacement vector of the point \( P(r_0, \theta_0, z_0) \) of the real component. It has been assumed that \( \mathbf{u} = \mathbf{u}_2 (r_0, z_0) + \frac{\theta_0}{\pi} \mathbf{u}_1 (r_0, z_0) \).
The stress/strain curve of the rivet’s material has been determined with a set compressive tests.

The interface conditions between the models’ components have been the following:

Figure 13. The interface conditions at the beginning of the simulation of the riveting process

The models have been loaded and constrained as shown in the following figure:
Figure 14. The load condition for the simulation of the riveting process

Working conditions

The fatigue life has been computed taking into account a mission profile whose duration is \( \Delta t = 500 \) h. The following quantities have then been defined:

- \( \Delta t_t \equiv \text{fraction of } \Delta t \text{ during which the engine runs at max torque} \)
- \( \Delta t_p \equiv \text{fraction of } \Delta t \text{ during which the engine runs at max power} \)
- \( \Delta t_o \equiv \text{fraction of } \Delta t \text{ during which the engine runs at overspeed} \)
- \( f_t \equiv \text{engine rotation frequency at max torque} \)
- \( f_p \equiv \text{engine rotation frequency at max power} \)
- \( f_o \equiv \text{engine rotation frequency at overspeed} \)

The standard Piaggio mission profile for engine bench tests has then been taken into account:

\[
\begin{align*}
\Delta t_t &= 0.05 \Delta t \\
\Delta t_p &= 0.85 \Delta t \\
\Delta t_o &= 0.1 \Delta t
\end{align*}
\]

During \( \Delta t \), the engine’s number of rotations is \( N_t = f_t \Delta t_t \). Similar considerations apply to \( \Delta t_p \) and \( \Delta t_o \). So the total number of rotations is

\[
N = f_t \Delta t_t + f_p \Delta t_p + f_o \Delta t_o = (0.05 f_t + 0.85 f_p + 0.1 f_o) \Delta t.
\]
Multiaxial stress fields have been reduced to uniaxial ones by means of the signed Von Mises stress, in order to carry out fatigue calculations based on data derived from uniaxial tests (Wöhler curves). The Goodman correction has been used for the mean stress effect, with no correction for negative stress states.

The same external loads and constraints have been used as for the simplified comparison. Additionally, the rivets’ temperature has been lowered, to make the rivets exert a compressive force on the flange and the pulley. The temperature decrease has been chosen to generate the compressive load calculated by the simulation of the riveting process. Frictional contact pairs were defined at the rivet/flange and rivet/pulley interfaces shown in the following figure:

![Figure 15. Interface surfaces between the rivets, the pulley and the flange](image)

Let $S_F$ and $S_P$ be the interface surface shown in the following figure:
Figure 16. Naming of interface surfaces between the rivets, the pulley and the flange

By means of the simulation of the riveting process, the pressure distributions at $S_F$ and $S_P$ were computed, due to the deformation of the rivet. Since two separate models have been used to simulate the riveting process, the four pressure distributions have been computed which are shown in the following figure:

Figure 17. The pressure distributions at the end of the riveting process, in the models used for the simulation of the riveting process
In the above figure the coordinate system and some values (a, b, c, d) of the z coordinate are shown.

In the model for the simulation of the working conditions, the pressure distributions have been simulated by means of an initial penetrations at SF and SP. Those penetrations generate two pressure distributions \( \hat{p}_F = \hat{p}_F(r, \theta, z) \) and \( \hat{p}_P = \hat{p}_P(r, \theta, z) \). The penetration values have been chosen to obtain

\[
\int_c^d \int_0^{2\pi} \hat{p}_F R d\theta dz = \frac{2\pi R \int_c^d \hat{p}_F dz + 2\pi R \int_c^d \hat{p}_F dz}{2}
\]

\[
\int_a^b \int_0^{2\pi} \hat{p}_P R d\theta dz = \frac{2\pi R \int_a^b \hat{p}_P dz + 2\pi R \int_a^b \hat{p}_P dz}{2}
\]

**Bench test simulation**

According to the Piaggio experimental procedures the flange/pulley/rivets system must undergo a bending bench test, simulating the bending effect induced by the belt’s tension, shown in the following figure:

![Figure 18. The belt's tension](image)

The belt needs to be tensioned because it transmits the motion by means of friction forces only.

A bending moment is applied to the flange, while the pulley is clamped along a circle at the distance from the rotation axis shown in the following figure:
The same constraint condition has been used in the model. In the model, a bending moment has been applied to the flange’s surface shown in the following figure:
The bending moment’s value has been derived from the previous simulation of the working conditions, during which the surface shown in the above figure has been clamped, instead of being loaded. The reactive bending moment has been considered, choosing the working condition which induced the largest reactive bending moment. That moment has been applied to the surface shown in the above figure, in the model for the simulation of the bending bench test.

Analysis Results & Discussion

**Simplified comparison**

The most stressed zones of the pulley have turned out to be subject to a tensile stress field. So simplified fatigue calculations have been carried out reducing the multiaxial stress state to an uniaxial one by means of the first principal stress.

A cylindrical coordinate system has been defined:

![Figure 21. The cylindrical coordinate system](image)

Let \(\sigma_1(r,\theta,z)\) be the first principal stress. The following quantities have been defined:

\[
\begin{align*}
\sigma_{\text{MAX}} &\equiv \max \{\sigma_1\} \\
\sigma_{\text{MIN}} &\equiv \min \{\sigma_1\} \\
\sigma_{\text{MAX}} \equiv r \mapsto \sigma_1 = \sigma_{\text{MAX}} \\
\theta_{\text{MAX}} \equiv \theta \mapsto \sigma_1 = \sigma_{\text{MAX}} \\
z_{\text{MAX}} \equiv z \mapsto \sigma_1 = \sigma_{\text{MAX}} \\
\sigma_{\text{MIN}} &= \min_\theta \sigma_{\{r_{\text{MAX}}, \theta, z_{\text{MAX}}\}}
\end{align*}
\]

\(\sigma_{\text{MIN}}\) and \(\sigma_{\text{MAX}}\) have been assumed to be the limits of the fatigue cycle the half-pulley is subject to.

Four design variants have been analysed and the one with the largest safety factor vs. the endurance limit has been singled out. The design variables have been the material and the geometry of the critical area, that
is, the area near the rivets (see the figure *the critical zones*). The material has been chosen among some aluminium alloys.

On the best configuration the effect of the number of rivets has been analysed.

Two possible solutions have been analysed: 8 or 10 rivets. The attention has been focused on the path shown in the following figure:

![Belt contact area](image)

**Figure 22. The path for the analysis of the effect of the number of rivets.**

That path has been chosen because of the position of the failures (see figure *The critical zones*). The curvilinear abscissa $s$ has been defined along the path.

The Von Mises stress has been mapped onto the path shown, for each of the three reference conditions. The results for the 8-rivets and 10-rivets configurations are shown in the following figure:
Figure 23. Comparison between the 8- and 10-rivets configurations

The above figure shows that the two configurations are nearly equivalent as far as the number of rivets is concerned.

Fatigue analysis

The riveting process

For each axisymmetric model shown in the figure Simulation of the riveting process, the pressure distribution has been computed between the pulley and the flange, as shown in the following figure:
Figure 24. The pressure distributions due to the riveting process

The force the rivets exert on the flange and the pulley has been computed as

\[
F = \frac{2\pi \int_{R_1}^{R_2} q_1rdr + 2\pi \int_{R_2}^{R_3} q_2rdr}{2}
\]

The force has turned out to be a little less than 9000 N.

The following figures show the distribution of the signed Von Mises stress on the surface of the pulley, in each of the two axisymmetric models used to simulate the riveting process.

Figure 25. The distribution of the signed Von Mises stress on the surface of the pulley at the end of the riveting process

The above figure shows that the part of the pulley near the external portion of the rivet seat (Part #2) is subject to a compressive stress field, which doesn’t worsen the fatigue conditions subsequently induced by the working conditions. The opposite can be observed for the internal side of the rivet seat, that is just the area where failure happened.
The working conditions

Each component of the stress tensor at the generic position in the pulley can be split as follows:

\[ \sigma_{ij} = \sigma_{ij}^{(w)} + \sigma_{ij}^{(r)} \]

where

\[ \sigma_{ij}^{(w)} \equiv \text{tension induced by the external loads}; \]

\[ \sigma_{ij}^{(r)} \equiv \text{residual stress at the end of the riveting process}. \]

The signed Von Mises stress values \( \sigma_{VM}^{(w)} \) and \( \sigma_{VM}^{(r)} \) have been computed from the tensors.

The models used to analyse the working conditions yield incorrect information about \( \sigma_{ij}^{(r)} \), since the interface conditions between the rivets and the pulley and between the rivets and the flange have been simulated in such a way that congruence exist \textit{only at an integral level, that is, only as for interface forces}, exploiting the results obtained with the simulations of the riveting process.

So only the values of \( \sigma_{VM}^{(w)} \) were taken from the models used to simulate the working conditions, subsequently adding them to the values of \( \sigma_{VM}^{(r)} \) obtained from the models used to simulate the riveting process.

So the results about \( \sigma_{ij}^{(w)} \) will be reported in the rest of this paragraph, except when explicitly stated.

The fatigue calculations have been carried out at the following locations in the pulley:

- **point A** → point where \( \sigma_{VM}^{(r)} \) attains its maximum value
- **point B** → point where \( \sigma_{VM}^{(w)} \) attains its maximum value under max torque conditions
- **point C** → point where \( \sigma_{VM}^{(w)} \) attains its maximum value under max power conditions
- **point D** → point where \( \sigma_{VM}^{(w)} \) attains its maximum value under overspeed conditions

The following figure shows the distribution of \( \sigma_{VM}^{(w)} \) under max torque conditions:
The path $s$ has been defined that is shown in the following figure.
The points $B_i$ in the above figure depict the positions point B reaches after each 36 degree rotation relative to the belt/pulley interface surface. Therefore, recording the $\sigma_{VM}^{(w)}$ value at the $B_i$ positions, the stress cycle can be computed during a full rotation of the pulley. That cycle will be characterized by an amplitude $a \sigma_{VM}^{(w)}$ and by a mean value $m \sigma_{VM}^{(w)}$. The real mean value, taking into account the residual stress field at the end of the riveting process, is therefore $m \sigma_{VM} = m \sigma_{VM}^{(w)} + \sigma_{YM}^{(r)}$ and has been used to compute the mean stress correction.

The residual stress value at point B after the riveting process has been obtained from the results of the simulation of the riveting process, as shown in the following figure:

**Figure 28. Combining the results from the simulations of the working conditions and the riveting process**

The same procedure has been used to compute the fatigue cycles under max power and overspeed conditions, determining the points C and D (see their definitions, given above). Point A has been determined from the model used to simulate the riveting process. Those points are shown in the following figure.
Figure 29. The critical points

The Goodman correction allowed to compute the equivalent stress amplitude at each point. The mission profile (see previous paragraph) allowed to compute the number of cycles the component undergo for each working condition. Therefore the cumulative damage for each condition could be computed, using Miner’s rule. The max power condition turned out to be the critical one.

The total damage was less 1, so the best configuration, chosen by the simplified comparison, proved to be able to withstand the mission profile.

Bench test simulation

The following figure shows the comparison between the Von Mises stress distributions obtained with the maximum torque working condition and bench test simulations:
Figure 30. Comparison between the Von Mises stress distributions obtained with the maximum torque working condition and bench test simulations

The stress distributions show their maximum values in the same region of the pulley., but the bench test generates a stress field which has a higher degree of symmetry with respect to the plane shown in the following figure:

Figure 31. Symmetry plane

As noticed in the previous paragraph, the critical working condition is at maximum power. The following figure shows a comparison between the Von Mises stress distribution at maximum power and the one induced by the bench test conditions:
Figure 32. Comparison between the Von Mises stress distribution at maximum power and the one induced by the bench test conditions

Although the most stressed points are near, they aren’t coincident.

So the bench test has proved to be appropriate to simulate the max torque condition, but not the max power one, which had turned out to be the critical one.

Conclusion

By means of a set of simplified, comparative analyses the best configuration has been singled out among different design variants.

The riveting process has been simulated by means of a simplified approach.

The best configuration has been analysed under the working conditions, including the effect of the riveting process. The configuration proved to be subject to a stress field under the endurance limit.

The standard bench test proved to be appropriate to reproduce only one of the reference working conditions.