With recent improvements in the efficiency of integral equation solutions it is now possible to combine the integral equation procedure with the finite element method (FEM) in a hybrid Finite Element Boundary Integral approach (FEBI) [1]. In electromagnetics the FEM is a general purpose technique that solves for volumetric electric fields and can be used to accurately characterize microwave components, antennas and signal integrity issues [2, 3]. For antenna or scattering problems, the air region surrounding the model must be included and terminated with an absorbing boundary condition (ABC). The integral equation (IE) approach, which solves directly for currents on object surfaces is not as general as the FEM, but is often more efficient for large open problems since it does not require the surrounding air volume. By combining the two approaches in a FEBI technique HFSS users can accurately and efficiently simulate very complex, large open problems. This new technique will be available in the 13.0 release of HFSS™.

One of the benefits of this technique is in reducing the size of the FEM domain of a given problem. The reflector antenna shown in Figure 1a is an example of the efficiency that can be realized with the FEBI approach. This is a 25λ (where λ is wavelength) diameter reflector excited by a dual mode circular waveguide feed itself supported by three struts. To simulate this using the FEM with a traditional ABC would require a large, convex and fully encompassing surrounding air volume. Instead a closely spaced conformal bounding air volume was placed around the model with the FEBI boundary applied to its surface as shown in the figure. The simulation of this electrically large antenna used approximately 10 GB RAM, and the resulting 3D radiation pattern is shown in Figure 1b. This is a significant result. A complex antenna that was once considered to be too large to be accurately modeled can now be simulated on a 12GB desktop computer.

![Figure 1 (a) 25λ reflector antenna. Feed is circular waveguide. The FEBI boundary is highly conformal and is shown. (b) 3D polar plot of radiation pattern in vicinity of main beam.](image-url)

This paper will discuss the application and advantages of FEBI for solving antenna and scattering problems. A set of benchmarks that demonstrate the accuracy and efficiency of this newly implemented procedure will be presented. Finally 2 examples that use the domain decomposition method (DMM) [4] together with the FEBI will be discussed.
OVERVIEW OF SIMULATION PROCEDURES

HFSS uses the FEM procedure to calculate the electric field (E). The solution space is discretized into geometrically conformal tetrahedral finite elements where the E field within each element is approximated by a local function. These approximate functions are substituted into the differential form of Maxwell’s equations. By representing the fields in this manner, Maxwell’s equations are transformed into a matrix equation. The resulting matrix equation is sparse as each element is only directly coupled to its neighboring elements and is solved using an efficient sparse matrix solver. Since the electric field is being computed the solution region must include that part of the model where the electric fields are present. Therefore, for antennas one must include the space around the device. Since the fields cannot be simulated out to infinity, the solution region around the radiating device must be terminated with an absorbing boundary condition that absorbs the radiated energy. With the proper ABC, the simulated device behaves as if it were operating in open space. Choices currently available in HFSS include a first order ABC where the boundary can be made to conform to the device. This ABC must be convex and should be spaced at least λ/4 [5]. The second alternative is the perfectly matched layer or PML. The PML is a better absorber and therefore a better representation of open space but is most easily assigned to a cubic boundary which may not be the most efficient with respect to minimizing the solution volume. A major advantage of the FEM is its robustness when modeling geometrically complex models that include complex dielectric structures. Another advantage of FEM is the option to use a wide variety of transmission line feeds as excitations whereby a 2D solver computes the proper modes for the given transmission line cross-section and then applies these fields to the 3D model using the transfinite element procedure [6].

In contrast to FEM, the integral equation (IE) procedure, available in HFSS-IE, computes the equivalent currents that are induced on the surface of objects in the model. For conducting objects the electric currents are computed and for finite dielectric objects the equivalent electric and magnetic currents are computed. This is often referred to as the Method of Moments (MoM). Basis functions that approximate the equivalent currents are placed on the surfaces of the objects. The appropriate boundary condition on each object is applied which results in an equation that includes all interaction between the various current elements. The resulting equations are tested with an appropriate set of weighting functions. While there are fewer unknowns for the same volumetric simulation with this procedure when compared to the FEM the resulting matrix is dense and as such its solution is not as efficient. In addition with this solution procedure the number of excitation choices is limited.

With the FEBI procedure the ABC in an FEM simulation is replaced by an integral equation based solution for computing directly the equivalent surface currents on the outer boundary of the FEM solution. This procedure starts by first computing the fields on the bounding surface of the FEM region using a first order ABC. This information is passed to the IE solver which computes a correction to these boundary fields. The FEM is then used to re-compute the fields with this new information. These steps are repeated until a converged response is found. Since the complete interaction between all elements on this boundary are computed using the IE solver the boundary can be moved closer to the model and can be made arbitrary, including concave, in shape with no loss in accuracy.

Based on this discussion it can be seen that the new FEBI boundary is the perfect combination of the FEM and IE solution techniques. It allows the more general FEM solver to be efficiently applied to large open problems by using an IE solver to reduce the solution region. The FEBI boundary can be placed on closely spaced conformal, including concave, radiation boundaries. Testing has shown a spacing of λ/10 is optimum. Given that both procedures are used in a FEBI simulation it will in general be less efficient than an equally sized FEM model with a first order ABC. Since it has FEM as its core, it can also take advantage of new features available in HFSS. This includes the recently introduced domain decomposition (DDM) method. For geometries that contain large dielectric, i.e. finite element regions, the DDM can be used together with the FEBI to efficiently simulate the model. In a DDM simulation, the solution region is broken into smaller domains and each domain solved using a separate computer core. Once the separate domains have been solved an iterative procedure is used to calculate the complete response. This allows the user to take advantage of networked memory to simulate larger models. As expected the memory per core should be smaller than the memory for the total model. Since the solver is operating on a smaller matrix for each domain it is often found that the total memory used is less than the total memory required to simulate the complete model on a single machine without DDM [7].
BENCHMARKS

For the first benchmark the accuracy of the radiated field calculation for a model with the FEBI applied to separate air regions will be considered. The example is a flared horn antenna spaced 7.8 \( \lambda \) above a 10 \( \lambda \times 10 \lambda \) flat conducting plate. This model can be simulated using HFSS with a PML (Figure 2).

Since there is a large air space between the source and the scatterer this model can also be split into two separate air volumes and simulated using HFSS with the FEBI. The model was simulated using both procedures and the resulting shade plots of the magnitude of the E field for the two simulations are shown in Figure 3.

Despite the two FEBI regions being separate, the fields in the vicinity of the plate are correctly computed. The radiation patterns in the YZ plane computed using both approaches for the plate and antenna combination are shown in Figure 4. The accuracy of the new approach is excellent. In addition to accurately computing the scattered fields the FEBI approach used less than 10% of the memory that the FEM with PML simulation required.

![Figure 2](image1.png)  
**Figure 2** FEM model of a flared horn illuminating a flat PEC plate. The PML layers are not shown.

![Figure 3](image2.png)  
**Figure 3** Shade plot of E field for (a)FEBI simulation with separate air volumes and (b) FEM model using PML.

![Figure 4](image3.png)  
**Figure 4** Comparison of \( \phi=90^\circ \) radiation pattern for model of Figure 2 computed using FEBI (solid) and FEM with PML (dashed).
As mentioned in the previous section the FEBI allows for concave boundaries. For the second benchmark, the scattering from a concave object will be considered. The object chosen is a corner reflector. The monostatic RCS for a large corner reflector was considered by Balanis et al in reference 8. In that paper are presented measured results for the scattering from a 90° corner that has sides that are 5.6088\(\lambda\) wide and 5.6088\(\lambda\) high. The HFSS model using the FEBI boundary is shown in Figure 5.

![Corner reflector model showing concave FEBI boundary.](image)

The air space around the reflector is concave. For this geometry it is important to include the higher order scattering between the two plates to be able to accurately characterize the backscatter. The monostatic RCS computed using the FEBI is compared to the measured data from reference 8 in Figure 6.

![RCS in \(\theta=90^\circ\) plane for corner reflector of Figure 4. FEBI computed results compared to measured results](image)

Once again the agreement is excellent. It can be seen that having a concave FEBI boundary does not affect the accuracy of field computation.

For the last benchmark the scattering from a coated test object will be considered. The object is shown in Figure 7.

![PEC cylindrical geometry coated with two layers of lossy dielectric.](image)

It is a 180cm long circular cylinder of radius 10 cm with spherical end caps. This PEC cylinder is coated with two lossy dielectric layers. Each layer is 0.75cm thick. The outer layer has \(\varepsilon_r=4-1j\) while the inner layer has \(\varepsilon_r=2.5-0.5j\). HFSS with the FEBI was used to simulate this model using a conformal air region that has a spacing of \(\lambda/10\). The monostatic RCS was computed at 1 GHz. The scattering from this object was also studied by Jin in Reference 9. In that reference a 2D body of revolution (BOR) code was used to compute the scattering. The FEBI data is compared to this BOR data in Figure 8 again with good agreement.
Figure 8  RCS for geometry shown in Figure 7. FEBI computed results are compared with computed data from Reference 9.

The 3 examples presented in this section confirm the accuracy of the new FEBI solution technique. Closely spaced, separate and concave boundaries were applied to antenna and scattering problems and in all cases accurate results were found.

**FEBI WITH DDM**

For the next example the RCS from a capped Luneburg lens will be computed. Capped dielectric lenses are used as radar reflectors. The lens modeled is shown in Figure 9a.

![Figure 9a](image)

![Figure 9b](image)

**Figure 9**  (a) Dielectric lens with PEC cap  (b) Plot of RCS.

For a Luneburg lens the shape is spherical with material properties that are a function of radius from the center. The materials are defined to have \( \varepsilon_r = 2 - (r/a)^2 \), where \( a \) is the radius of the lens. The lens chosen here is \( 10\lambda_0 \) in diameter. Creating such a spatially dependent material is difficult in practice. To approximate the lens, a set of 5 concentric homogeneous spheres are used. The bistatic RCS data is shown in Figure 9b. Simulating the full model would require a computer with >20GB memory. Different element orders, solvers or the use of symmetry can be applied to reduce the problem to a smaller size or the domain decomposition method (DDM) introduced in HFSS 12 can be used. In the DDM the solution space is broken up into a set of smaller finite element domains. When FEBI is implemented with the DDM one domain will contain the entire FEBI boundary. To demonstrate the effectiveness of using the FEBI and DDM together this project was run using different numbers of domains.
In Figure 10 is a plot of the average memory used per domain versus the number of domains. A plot of what the memory per domain would be if the memory scaled linearly from the non-DDM simulation is also included for comparison. Since the solver is operating on a smaller matrix in each domain, the memory used per domain decreases at a rate greater than linear.

*Figure 10 Memory per domain – plot is log-log.*

To further illustrate the effectiveness of the FEBI and DDM combination, the two will be used together to simulate an antenna mounted on a complex platform. The antenna is shown in Figure 11a. It is a 7 element array of helix antennas operating at 3.5 GHz. The array is mounted on a satellite platform. Many of the parts of the satellite platform are dielectric objects, so this is not an ideal project for simulation using HFSS-IE. This model could be analyzed using the standard ABC and a properly spaced rectangular air box, but the volume enclosed in the solution region would be very large. A simulation of this model using the standard ABC and the DDM alone needed 210 GB of total memory [10]. Utilizing the FEBI instead, a highly conformal air volume can be used. A conformal air volume was created and is shown in Figure 11b. The total memory needed to simulate this new model using the FEBI and DDM was only 21 GB [10]. By combining the FEBI with the DDM the memory needed to simulate this complex mounted antenna model was reduced by a factor of 10. The 3D radiation pattern is shown in Figure 12.

*Figure 11 (a) 7 element array mounted on a satellite platform. (b) surrounding air volume with FEBI boundary shown.*

*Figure 12 Radiation pattern for the mounted antenna array shown in Figure 11.*
CONCLUSION
The new FEBI technique gives the design engineer the advantages of an FEM simulation with the efficiency and accuracy of an IE solution for open boundary problems. This procedure is accurate for conformal, concave and/or separate air volumes, allowing users to reduce the size of the solution region. Reducing the problem size results in a significant reduction in the solution time and the amount of memory required to solve the problem. How does one chose which approach to use? A few basic guidelines can be derived from the results shown here.

In general the IE solver is optimal for simulating large structures that are mostly conducting. For example: RCS from large scatterers and antennas mounted on conducting structures such as a GPS antenna on an automobile. When the FEM is appropriate it is best to use a FEBI boundary over the PML or ABC when doing so would eliminate a large volume from the solution region. This was shown in the benchmark examples of the horn above the plate and the corner reflector where use of the FEBI allowed large volumes with average dimensions on the order of 5-10λ to be eliminated. This reduced the memory requirements significantly. A few examples where such reductions are possible are: antennas and scatterer combinations such as reflector antennas, concave structures such as a reflector or an aircraft body, and antenna structures where separate air regions can be used such as an antenna near a scatterer or the coupling between multiple antennas.
REFERENCES


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